Sheet 6

P16

a)

$$H_0(x) = e^{x^2} e^{-x^2} = \underline{\underline{1}}$$

$$H_1(x) = (-1)\, e^{x^2} \frac{d}{dx} e^{-x^2}$$

$$= (-1)\, e^{+x^2} (-2x)\, e^{-x^2}$$

$$= \underline{\underline{2x}}$$

$$H_2(x) = e^{x^2} \frac{d^2}{dx^2} e^{-x^2} = e^{x^2} \frac{d}{dx}\left(-2x\, e^{-x^2}\right)$$

$$= e^{x^2}\left(-2\, e^{-x^2} + (-2x)(-2x)\, e^{-x^2}\right)$$

$$= \underline{\underline{4x^2 - 2}}$$

b)

$$\int_{-\infty}^{\infty} H_0(x) H_1(x) e^{-x^2} dx = \int_{-\infty}^{\infty} \underbrace{2x}_{\text{anti-symmetric}} \underbrace{e^{-x^2}}_{\text{symmetric}} dx = 0$$

$$\int_{-\infty}^{\infty} H_1(x) H_1(x) e^{-x^2} dx = \int_{-\infty}^{\infty} 4x^2 e^{-x^2} dx = 4 \cdot \frac{\sqrt{\pi}}{2} = 2\sqrt{\pi}$$

$$\int_{-\infty}^{\infty} H_1(x) H_2(x) e^{-x^2} dx = \int_{-\infty}^{\infty} \underbrace{2x}_{\text{antisymmetric}} \cdot \underbrace{(4x^2-2)\, e^{-x^2}}_{\text{symmetric}} dx = 0$$

P17

a)

$$E_R = E_0 \sqrt{R} \cdot \left[1 - (1-R) e^{i\Delta\varphi} \sum_{m=0}^{\infty} R^m e^{im\Delta\varphi}\right]$$

with $q = R \cdot e^{i\Delta\varphi}$ we obtain

$$E_R = E_0 \sqrt{R} \cdot \left[1 - (1-R) e^{i\Delta\varphi} \sum_{m=0}^{\infty} q^m\right]$$

$$= E_0 \sqrt{R} \cdot \left[1 - (1-R) e^{i\Delta\varphi} \frac{1}{1-q}\right]$$

$$= E_0 \sqrt{R} \cdot \left[1 - \frac{(1-R) e^{i\Delta\varphi}}{1 - R e^{i\Delta\varphi}}\right]$$

$$= E_0 \sqrt{R} \cdot \frac{1 - \cancel{R e^{i\Delta\varphi}} - e^{i\Delta\varphi} + \cancel{R e^{i\Delta\varphi}}}{1 - R e^{i\Delta\varphi}}$$

$$= E_0 \sqrt{R} \frac{1 - e^{i\Delta\varphi}}{1 - R e^{i\Delta\varphi}}$$

$$I_R \propto |E_R|^2 \Rightarrow I_R = I_0 R \cdot \left| \frac{1 - e^{i\Delta\varphi}}{1 - R e^{i\Delta\varphi}} \right|^2$$

$$= I_0 R \cdot \frac{|1 - e^{i\Delta\varphi}|^2}{|1 - R e^{i\Delta\varphi}|^2} = I_0 R \cdot \frac{(1 - e^{i\Delta\varphi})(1 - e^{-i\Delta\varphi})}{(1 - R e^{i\Delta\varphi})(1 - R e^{-i\Delta\varphi})}$$

$$= I_0 R \cdot \frac{1 - e^{-i\Delta\varphi} - e^{i\Delta\varphi} + 1}{1 - R e^{-i\Delta\varphi} - R e^{i\Delta\varphi} + R^2}$$

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$$\Rightarrow \quad I_R = I_0 R \cdot \frac{2 - 2\cos\Delta\varphi}{1 + R^2 - 2R\cos\Delta\varphi} \qquad \text{with } 1 - \cos x = 2\sin^2\frac{x}{2}$$

we obtain

$$I_R = I_0 R \cdot \frac{4\sin^2\frac{\Delta\varphi}{2}}{(1-R^2) + 4R\sin^2\frac{\Delta\varphi}{2}} = I_0 \frac{A\sin^2\frac{\Delta\varphi}{2}}{1 + A\sin^2\frac{\Delta\varphi}{2}} \qquad \text{qed}$$

$I_T = 1 - I_R$ as no absorbtion or losses occur.

$$\Rightarrow \quad I_T = 1 - I_R = 1 - I_0 \frac{A\sin^2\frac{\Delta\varphi}{2}}{1 + A\sin^2\frac{\Delta\varphi}{2}} = I_0 \frac{1}{1 + A\sin^2\frac{\Delta\varphi}{2}} \qquad \text{qed.}$$

b) The transmission maxima $I_T \to max$ occur ~~at~~ at $\frac{\Delta\varphi}{2} = j \cdot \pi$

$$\Rightarrow \quad \pi \frac{\Delta s}{\lambda} = j \cdot \pi \quad \Rightarrow \quad \frac{\Delta s}{\lambda} = j$$

$$\Rightarrow \quad \lambda_{max,j} = \frac{\Delta s}{j} = \frac{2d}{j}\sqrt{n^2 - \sin^2\alpha} \qquad \text{qed.}$$

c) The transmission minima occur at $\frac{\Delta\varphi}{2} = (2j+1)\frac{\pi}{2} \quad \Rightarrow \quad \frac{\Delta s}{\lambda} = \frac{2j+1}{2}$

$$\Rightarrow \quad \lambda_{min,j} = \frac{4d}{2j+1}\sqrt{n^2 - \sin^2\alpha} \qquad \text{qed.}$$

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P18

a) $\frac{2L}{\lambda_{oog}} = q + \frac{1}{2}$

$\Rightarrow q = \frac{2L}{\lambda_{oog}} - \frac{1}{2} = \frac{2 \cdot 300mm}{632.8nm} - \frac{1}{2} = 948166$

b) $P_{out} = 10mW$

$\Rightarrow \frac{1}{2} P_{tot} \cdot (1 - R_{oc}) = P_{out}$

$\Rightarrow P_{tot} = \underline{\underline{1W}}$

Number of photons : $\Phi = \frac{\lambda_s}{hc^2} \cdot I_s \Rightarrow N_p = \frac{\lambda_s \cdot I_s}{hc^2} \cdot V = \frac{\lambda_s}{hc^2} \cdot I_s \cdot A \cdot L$

$= \frac{\lambda_s}{hc^2} P_{tot} \cdot L$

$\Rightarrow N_p = \frac{0.3m \cdot 632.8nm}{hc^2} \cdot 1W$

$= \underline{\underline{3.19 \cdot 10^3}}$

$P_{tot} = P_c + P_{sp}$ and $\frac{P_c}{P_{sp}} = N_p$

$P_{tot} = P_{sp}(N_s + 1)$

$\Rightarrow P_{sp} = \frac{P_{tot}}{N_p + 1} = \underline{\underline{3.14 \cdot 10^{-10} W}}.$

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P18 c)

$$\tau_c = -\frac{2L}{c \ln\left[R_{oc}(1-\Lambda)\right]} \qquad \text{with} \quad R_{oc} = 0.98 \quad \text{and} \quad \Lambda = 0.004$$

$$\Rightarrow \quad \underline{\underline{\tau_c = 82.7\,\text{ns}}}$$

$$\Rightarrow \quad \Delta\nu_c = \frac{1}{2\pi\tau_c} = 1.93\ \text{MHz}$$

$$\Rightarrow \quad \delta\nu_L \approx \frac{\Delta\nu_c}{N_p} = \frac{1.93\ \text{MHz}}{3.19 \cdot 10^9} = \underline{\underline{6 \cdot 10^{-4}\ \text{Hz}}}$$

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